

Lecture 3: Energy in Thermal Physics

Lecturer: Qiang Zhu

Scribes: scribe-name1,2,3

3.1 Some useful equations

$$PV = nRT = NkT \quad (3.1)$$

$$N = n \times N_A \quad (3.2)$$

$$PV = Nm\bar{v}_x^2 = NkT \quad (3.3)$$

$$U_{\text{thermal}} = N \cdot f \cdot \frac{1}{2}kT \quad (3.4)$$

How to count the number of degrees of freedom (f)

1. translation, rotation, vibration as a function of temperature
2. number of atoms in the molecule, monoatomic, diatomic, etc
3. internal symmetry

3.2 First law of thermodynamics

By conservation of energy, the change in total thermal energy is the sum of heat entering the system and work done on the system,

$$\Delta U = Q + W \quad (3.5)$$

Q: heat transfer by conduction, convection, radiation.

W: mechanical/electric/chemical work.

3.3 Compression Work and PV diagram

while we already know how to calculate U according to eq 3.4, let's try to figure out how to calculate W . We always start from its original definition,

$$\Delta W = F\Delta X \quad (3.6)$$

Suppose this is a quasistatic compression, i.e, every step is very slow and reversible, then we have

$$\Delta W = P\Delta X \quad (3.7)$$

for each step. By merging $A\Delta X$ term, we get

$$\Delta W = -P\Delta V \quad (3.8)$$

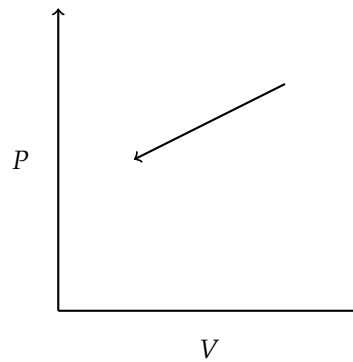
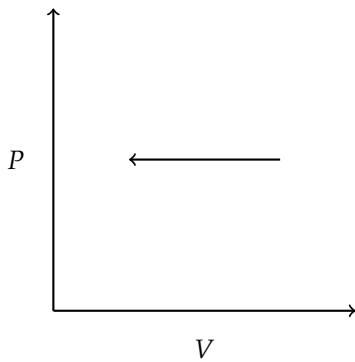
A simple way to check if the derived equation is correct. W and $-P\Delta V$ both have the unit of J. How to calculate $-P\Delta V$?

1. P is fixed during compression
2. P is not constant

We need to integrate each small step for eq.3.8, hence we have

$$W = - \int_{V_i}^{V_f} P(V)dV. \quad (3.9)$$

Graphically, it means the total area under the graph of P v.s. V .



Exercises

(Problem 1.32):

By applying a pressure of 200 atm, you can compress water to 99% of its usual volume. Sketch the process on a PV diagram, and estimate the work required to achieve it. Does the result surprise you?

(Problem 1.33): Analyze the cyclic process shown as follows,

	A	B	C	Total
W				
Q				
U				

3.3.1 Compression on ideal gas

Let's think about how compression is done on the ideal gas. There are two extremes as follows.

1. very slow that the temperature doesn't change at all, i.e., isothermal compression
2. very fast that the no heat escapes from the gas, i.e., adiabatic compression

What do they indicate:

1. if T doesn't change, U is constant.
2. if no heat escapes, Q is zero.

3.3.2 Isothermal Compression

Under isothermal conditions,

$$W = - \int_{V_i}^{V_f} P(V) dV = -NkT \int_{V_i}^{V_f} \frac{1}{V} dV. \quad (3.10)$$

Remember some special functions, e^x , $\ln x$, $\sin(x)$, $\cos(x)$.

$$W = -NkT(\ln V_f - \ln V_i) \quad (3.11)$$

Since U is constant, Q is simply $-W$.

3.3.3 Adiabatic Compression

Remember $U=W$ under adiabatic conditions, hence we have

$$dU = \frac{f}{2} Nk dT, \quad (3.12)$$

$$dW = -PdV. \quad (3.13)$$

Since $dU=dW$

$$-PdV = \frac{f}{2} Nk dT. \quad (3.14)$$

By plugging in $PV = NkT$, we get

$$-\frac{dV}{V} = \frac{f}{2} \frac{dT}{T}. \quad (3.15)$$

Let us integrate here,

$$\int_{V_i}^{V_f} -\frac{dV}{V} = \frac{f}{2} \int_{T_i}^{T_f} \frac{dT}{T}. \quad (3.16)$$

then we get

$$\ln \frac{V_f}{V_i} = \frac{f}{2} \ln \frac{T_i}{T_f}. \quad (3.17)$$

To remove the logarithmic sign, we have

$$V_f T_f^{f/2} = V_i T_i^{f/2} = \text{const.} \quad (3.18)$$

Alternatively, we can also rewrite it in the form of P instead of T ,

$$V^\gamma P = \text{const.} \quad (3.19)$$

where $\gamma = \frac{f+2}{f}$ is called the adiabatic exponent.

Homework: how to prove it? It will be intensively used in the following class!

