## Lecture 4: Energy in Thermal Physics

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### 4.1 Compression Work

$$
\begin{align*}
V T^{f / 2} & =\text { const. }  \tag{4.1}\\
V^{\gamma} P & =\text { const. } \tag{4.2}
\end{align*}
$$

where $\gamma=\frac{f+2}{f}$ is called adiabatic exponent.

## Exercises

(Problem 1.38):
Two identical bubbles rise from the bottom of a lake to its surface.
Bubble A rises quickly (adiabatic condition).
Bubble B rises slowly (isothermal condition).
Which buble is larger in the end?
(Problem 1.40):
In Lec02, we have determined that

$$
\begin{equation*}
\frac{d P}{d z}=-\frac{m g}{k T} P \tag{4.3}
\end{equation*}
$$

and obtained the pressure $P$ as a function of height as follows,

$$
\begin{equation*}
P(z)=P(0) \exp (-m g z / k T) \tag{4.4}
\end{equation*}
$$

1) show that $T$ and $P$ has the following relation

$$
\begin{equation*}
\frac{d T}{d P}=\frac{2}{f+2} \frac{T}{P} \tag{4.5}
\end{equation*}
$$

2) find a formula for $d T / d z$ like $d P / d z$.
3) estimate the temperature at the peak of Mount Everest $(8848 \mathrm{~km})$.

### 4.2 Heat Capacities

The heat capacity of an object is the amount of heat needed to raise its temperature,

$$
\begin{equation*}
C=\frac{Q}{\Delta T}=\frac{\Delta U-W}{\Delta T} \tag{4.6}
\end{equation*}
$$

Specific heat capacity is a more fundamental metric,

$$
\begin{equation*}
c=\frac{C}{m} . \tag{4.7}
\end{equation*}
$$

There are two types of circumstances

1. constant volume, $C_{V}$
2. constant pressure, $C_{P}$

Obviously, $C_{P}$ is more representative than $C_{V}$. According to the eqipartition theorem,

$$
\begin{equation*}
C_{V}=\frac{\partial U}{\partial T}=\frac{\partial}{\partial T}\left(\frac{N f k T}{2}\right)=\frac{N f k}{2} \tag{4.8}
\end{equation*}
$$

Under constant pressure,

$$
\begin{align*}
C_{P} & =\frac{\partial(U-W)}{\partial T}=\frac{\partial}{\partial T}\left(\frac{N f k T}{2}\right)-\frac{\partial W}{\partial T}  \tag{4.9}\\
\frac{\partial W}{\partial T} & =-P \frac{\partial V}{\partial T}=-P \frac{\partial(N k T / P)}{\partial T}=-N k \tag{4.10}
\end{align*}
$$

therefore,

$$
\begin{equation*}
C_{P}=C_{V}+N k \tag{4.11}
\end{equation*}
$$

using ideal gas law.

## Discussions:

1. What's the relation between $C_{V}$ and $C_{P}$ for solids?
2. $C_{V}$ of 1 mole of $\mathrm{H}_{2}$ gas as a function of temperature (see Figure 4.1)
3. $C_{P}$ of 1 mole of elemental solids as a function of temperature (see Figure 4.2)

## Exercises

(Problem 1.44): Look up the table of thermodynamic data at room temperature. Browse through the $C_{P}$ values in this table, and understand them according to the equipartition theorem.

| Type | Examples | Ideal Value | Anomaly |
| :---: | :--- | :--- | :--- |
| monoatomic gas |  |  |  |
| diatomic gas |  |  |  |
| polyatomic(linear) gas |  |  |  |
| polyatomic gas |  |  |  |
| Elemental solid |  |  |  |
| Binary solid |  |  |  |



Figure 4.1: Heat capacity at constant volume of one mole of hydrogen $\left(\mathrm{H}_{2}\right)$ gas. Note that the temperature scale is logarithmic.


Figure 4.2: Measured heat capacities at constant pressure (data points) for one mole each of three different elemental solids.

### 4.3 Enthalpy

Constant pressure processes occur quite often in chemical reactions and phase transformations. Keeping track of the work done during these processes gets to be a pain. Any idea to make it more convenient? Instead of talking about the energy, we can agree to add the work due to $P V$ in the given environment. This results in a new quantity called the enthalpy $(H)$,

$$
\begin{equation*}
H=U+P V \tag{4.12}
\end{equation*}
$$

Conveniently, we can express

$$
\begin{equation*}
C_{P}=\left(\frac{\partial H}{\partial T}\right)_{P} \tag{4.13}
\end{equation*}
$$

Think about

1. diamond and graphite under high pressure
2. formation enthalpy of $\mathrm{H}_{2}$ and $\mathrm{O}_{2}$ gas combine to water
