Physics 467/667: Thermal Physics **Spring 2019**

Lecture 5: The Second Law and Entropy

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5.1 Introduction

We have explored the law of energy conservation and applied it to thermodynmic systems. In the meantime, we studied the relations between heat, work and thermal energy, and the connections between marcoscopic observables P, V, T and miscroscopic properties v, f.

However, some very fundamental questions remain unanswered,

1. *what is temperature?*
2. *why does heat flow spontaneously from hot to cold objects?*
3. *why do many processes happen in one direction, but never the reverse?*

5.2 Combinatorics and Two-state systems

Let's get started with a silly example of flipping three coins. How many possible outcomes are there?

Coin1	Coin2	Coin3
H	H	H
H	H	T
H	T	H
T	H	H
H	T	T
T	T	H
T	H	T
T	T	T

In total we have 8 outcomes, each is called a **microstate**.
Since all coins are indistinguishable, we are more interested in how many heads are there in all outcomes?
By simply counting the number from the above table, we know
3 heads, HHH
2 heads, HHT, HTH, THH
1 head, HTT, TTH, THT
0 head, TTT

each state is called a **microstate**.
microstate: Each of the eight different outcomes
macrostate: How many heads are there in all outcomes

Although each microstate can equally exist, but macrostates have different probabilities to explored. Clearly, 2 heads is more likely to be found than 3 heads. Here we introduce another quantity,
multiplicity (Ω): the number of microstates in a given macrostate.

In the context of coin games, let's define $\Omega(n)$ as number of cases when we get n heads. If the total number of coins is N, we can derive the equations as follows,

$$\Omega(N,n) = \frac{N!}{n! \cdot (N-n)!} = \binom{N}{n} \tag{5.1}$$

What will happen if we increase N. Let's try N = 4 and 20 in Problem 2.1 and 2.2.

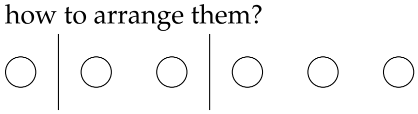
Such two-state systems are quite common in physics, such as two-state paramagnet.

5.3 The Einstein Model of a Solid

N: Number of oscillators.
q: Number of energy states.

$$\Omega(N,q) = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q!(N-1)!} \tag{5.2}$$

It can be simply proved as follows
q circles;
$N-1$ vertical lines;
how to arrange them?



Exercises
Calculate the multiplicity of an Einstein solid with 5 oscillators and [1,2,3,4,5] units of Energy.

q	$\Omega(5,q)$
1	
2	
3	
4	
5	

Computer programming

1. Write a small piece of program to calculate $\Omega(N)$ in the context of flipping coins and plot them when N=10, 15, 30, 100.

2. Write a small piece of program to calculate $\Omega(N)$ in the context of Einstein solid and plot them when N=10, 15, 30, 100 and q from 0 to 10.

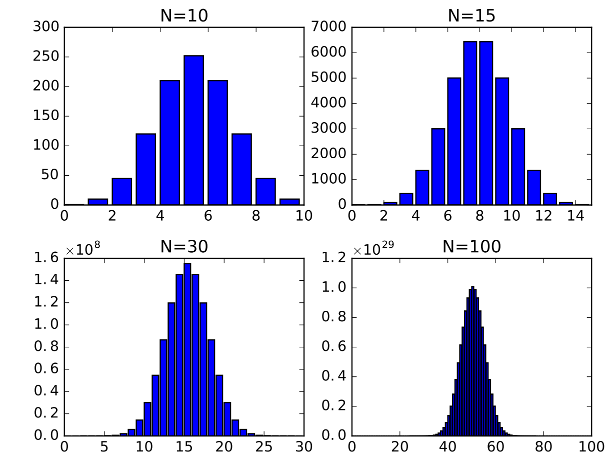


Figure 5.1: The value of Ω as a function of N in the game of flipping coins.

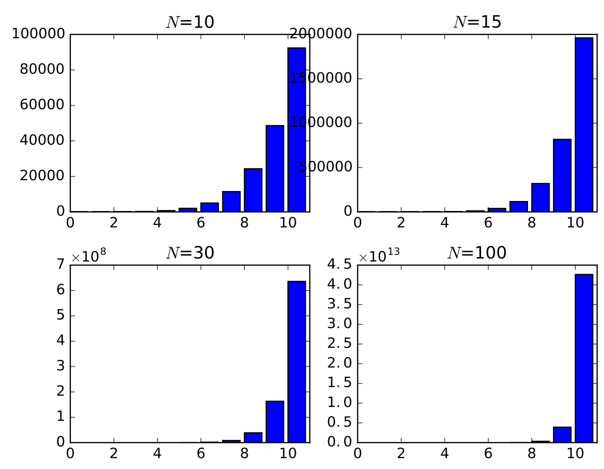


Figure 5.2: The value of Ω as a function of N in an Einstein solid.