Physics 467/667: Thermal Physics **Spring 2019**

Lecture 28: Bose-Einstein Condensation

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28.1 The statistical behavior for Bosons

As stated in the introduction to Fermions and Bosons, quantum statistics starts to play a role in dense system and low temperatures. For an atom at room temperature, the quantum volume is

$$\epsilon_0 = \frac{h^2}{8mL^2}(1^2 + 1^2 + 1^2) = \frac{3h^2}{8mL^2} \tag{28.1}$$

$$N_0 = \frac{1}{e^{(\epsilon_0 - \mu)/kT} - 1} \tag{28.2}$$

When T is very small, N_0 will be quite large. In this case, the denominator must be small,

$$N_0 = \frac{1}{1 + (\epsilon_0 - \mu)/kT - 1} = \frac{kT}{\epsilon_0 - \mu} \quad (\text{when } N_0 \gg 1) \tag{28.3}$$

The chemical potential μ must be equal to ϵ_0 at T=0, and just a bit less than ϵ_0 at small T. Now the question is **at which temperature we can observe that N_0 remains very large?**

28.2 Computing the total number of Bosons

$$N = \sum_s \frac{1}{e^{(\epsilon_s - \mu)/kT} - 1} \tag{28.4}$$

In practice, we can turn it to an integral,

$$N = \int_0^\infty g(\epsilon) \frac{1}{e^{(\epsilon_s - \mu)/kT} - 1} d\epsilon \tag{28.5}$$

Where $g(\epsilon)$ is the density of states, which has a similar form following the electron gas model.

$$g(\epsilon) = \frac{2}{\sqrt{\pi}} \left(\frac{2\pi m}{h^2}\right)^{3/2} V\sqrt{\epsilon} \tag{28.6}$$

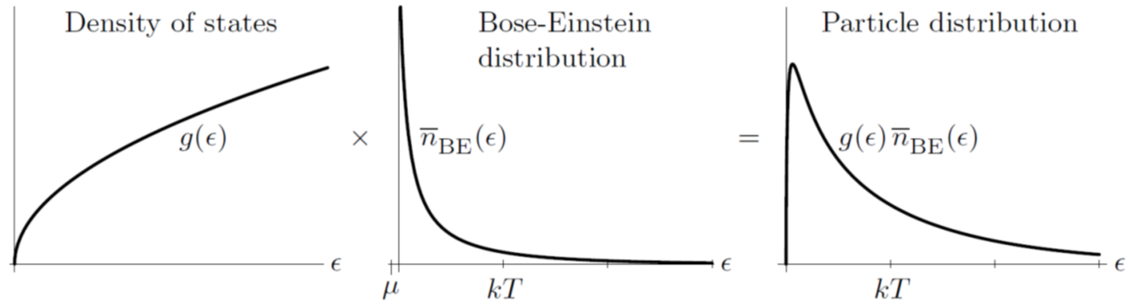

Figure 28.1: The distribution of bosons as a function of energy is the product of two functions, the density of states and the Bose-Einstein distribution. Copyright 2000, Addison-Wesley.

The trouble is that we cannot evaluate eq.(28.5) analytically. In order to work it out, we must guess some value for the μ term. A good starting point is let μ=0. Changing the variable to $x = \epsilon/kT$

$$\begin{aligned} N &= \frac{2}{\sqrt{\pi}}\left(\frac{2\pi m}{h^2}\right)^{3/2} V \int_0^\infty \frac{\sqrt{\epsilon}d\epsilon}{e^{\epsilon/kT}-1} \\ &= \frac{2}{\sqrt{\pi}}\left(\frac{2\pi mkT}{h^2}\right)^{3/2} V \int_0^\infty \frac{\sqrt{x}dx}{e^x-1} \end{aligned} \tag{28.7}$$

The integral over x gives 2.315, which leaves us with

$$N = 2.612\left(\frac{2\pi mkT}{h^2}\right)^{3/2} V \tag{28.8}$$

That result is wrong! It means that the number of particles purely depends on T. In fact, there can be only one T corresponds to this value.

$$N = 2.612\left(\frac{2\pi mkT_c}{h^2}\right)^{3/2} V \tag{28.9}$$

When $T < T_c$, this will be no longer valid during the transformation from summation to integral. This is because the terms from $\epsilon = 0$ are missing. Therefore, it should be better expressed as

$$N_{\text{exited}} = 2.612\left(\frac{2\pi mkT}{h^2}\right)^{3/2} V \tag{28.10}$$

While the gap between N and N_{exited} is the number of atoms in the ground state.

$$N_0 = N - N_{\text{exited}} = \left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right] N \tag{28.11}$$

The abrupt accumulation of atoms in the ground state below T_c is called **Bose-Einstein condensation**.

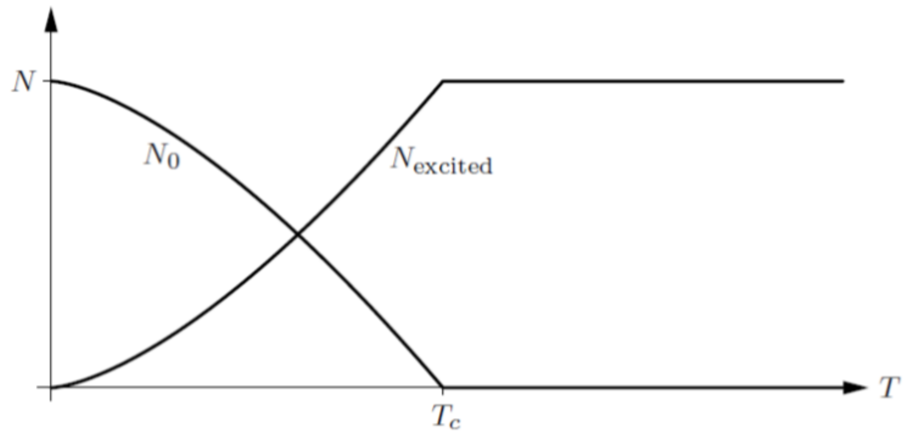

Figure 28.2: Number of atoms in the ground state (N_0) and in excited states, for an ideal Bose gas in a three-dimensional box. Below T_c the number of atoms in excited states is proportional to $T^{3/2}$. Copyright 2000, Addison-Wesley

28.3 Real World Examples

In 1995 BEC of a gas of weakly interacting atoms was first achieved using Rb-87. Later, BEC was achieved with dilute gases of atomic Li, Na, H, .etc.

The superfluid phase of ^{4}He is also considered to be an example of BEC.

28.4 Why does it happen?